



Example 1:

Rank the following in ascending order:

$$2^{120}, 3^{72}, 17^{30}$$

*Solution*

Comparing  $2^{120}$  and  $17^{30}$

HCF of 120 and 30 = 30

$$2^{120} = 2^{4 \times 30} = 16^{30}$$

Since  $17 > 16$

$$17^{30} > 16^{30}$$

Hence  $17^{30} > 2^{120}$  ... (1)

Comparing  $2^{120}$  and  $3^{72}$

HCF of 120 and 72 = 24

$$2^{120} = 2^{5 \times 24} = 32^{24}$$

$$3^{72} = 3^{3 \times 24} = 27^{24}$$

We know that  $27 < 32$

Hence  $27^{24} < 32^{24}$

Thus

$$3^{72} < 2^{120}$$
 ... (2)

From (1) and (2)

$$3^{72} < 2^{120} < 17^{30}$$

Example 2:

If  $K^{3/2}$  is 50% greater than  $K^{5/4}$ , what is the value of K?

*Solution*

From the given info.

$$K^{5/4} + 50\% \text{ of } K^{5/4} = K^{3/2}$$

$$3/2 * K^{5/4} = K^{3/2}$$

$$3/2 = K^{3/2} / K^{5/4}$$

$$3/2 = K^{3/2 - 5/4}$$

$$3/2 = K^{1/4}$$

$$3^4/2^4 = K$$

$$K = 81/16$$

Example 3:

$$\text{If } (25 \sqrt{5})^x = (\sqrt[3]{5})^{x+1}$$

Find x.

*Solution:*

$$25 = 5^2 \qquad \sqrt{5} = 5^{1/2} \qquad \sqrt[3]{5} = 5^{1/3}$$

Given equation can be written in the form:

$$(5^2 * 5^{1/2})^x = (5^{1/3})^{x+1}$$

$$(5^{2 + 1/2})^x = 5^{(x+1)/3}$$

$$5^{5x/2} = 5^{(x+1)/3}$$

Equating powers with same base

$$5x/2 = (x+1)/3$$

$$15x = 2x + 2$$

$$14x = 2$$

$$x = 1/7$$

Example 4:

$$\frac{4 + 2\sqrt{5}}{7 + 3\sqrt{5}} = ?$$

*Solution*

Rationalization

$$\frac{4 + 2\sqrt{5}}{7 + 3\sqrt{5}} * \frac{7 - 3\sqrt{5}}{7 - 3\sqrt{5}} = \frac{28 - 12\sqrt{5} + 14\sqrt{5} - 35}{49 - 45} = \frac{-7 + 2\sqrt{5}}{4}$$

Example 5:

What is the value of  $\sqrt{25 + 10\sqrt{6}} + \sqrt{25 - 10\sqrt{6}}$

*Solution:*

Assume



Cycle of 4

Units digit of  $3^9 =$  units digit of  $3^{4*2 + 1} = 3$

Since  $(17^3)^4$  is less  $1973^{3^2}$

Units digit of  $(17^3)^4 - 1973^{3^2}$  will be the units' digit of  $1973^{3^2} - (17^3)^4$

$$= 3 - 2 = 1$$

Example 7:

If  $5^{10x} = 4900$  and  $2^{\sqrt{y}} = 25$ . Find the value of:

$$\frac{(5^{(x-1)})^5}{4^{-\sqrt{y}}}$$

*Solution:*

Given expression:

$$\frac{(5^{(x-1)})^5}{4^{-\sqrt{y}}} = \frac{(5^x * 5^{-1})^5}{\frac{1}{4^{\sqrt{y}}}} = 4^{\sqrt{y}} * (5^{5x} * 5^{-1})$$

$$= 2^{2*\sqrt{y}} * \left(5^{\frac{10x}{2}}\right) * \frac{1}{5}$$

$$= (2^{\sqrt{y}})^2 * (5^{10x})^{\frac{1}{2}} * \frac{1}{5}$$

$$= 25^2 * 4900^{\frac{1}{2}} * \frac{1}{5}$$

$$= 625 * \frac{70}{5}$$

$$= 9000$$

Example 8:

If  $2^x - 2^{x-2} = 3 * 2^{13}$  find x

*Solution:*

$$2^x - 2^{x-2} = 3 * 2^{13}$$

$$2^x - 2^x * 2^{-2} = 3 * 2^{13}$$

$$2^x (1 - 1/2^2) = 3 * 2^{13}$$

$$2^x \cdot \frac{3}{4} = 3 \cdot 2^{13}$$

$$2^x = 4 \cdot 2^{13} = 2^2 \cdot 2^{13} = 2^{15}$$

$$x = 15$$